STOCHASTIC DOMINANCE-BASED COMPARISON BETWEEN ITERATIVE IMPROVEMENT AND SIMULATED ANNEALING ALGORITHMS FOR MINIMUM LOSSES DISTRIBUTION SYSTEMS RECONFIGURATION

COMPARAȚIE BAZATĂ PE DOMINANȚĂ STOCASTICĂ ÎNTRE ALGORITMII DE ÎMBUNĂTĂȚIRE ITERATIVĂ ȘI DE RECOACERE SIMULATĂ PENTRU RECONFIGURAREA SISTEMELOR DE DISTRIBUȚIE CU PIERDERI MINIME

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Abstract: *This paper presents a comparison between two different techniques used for solving the distribution systems reconfiguration problem with the aim of minimizing the power losses. The solution methods considered are the iterative improvement algorithm and the simulated annealing algorithm. The methods are applied to a reconfigurable distribution system considering the radiality and operational constraints. The comparison is carried out by resorting to a proposed variant of an indicator based on the first-order stochastic dominance.*

Keywords: distribution systems, reconfiguration problem, minimum power losses, iterative improvement, simulated annealing, stochastic dominance.

Rezumat: *Această lucrare prezintă o comparație între două tehnici diferite utilizate pentru rezolvarea problemei de reconfigurare a sistemelor electrice de distribuție, în vederea minimizării pierderilor de putere activă. Metodele de rezolvare luate în considerare sunt algoritmul de îmbunătățire progresivă și algoritmul de călire simulată. Metodele sunt aplicate unui sistem electric de distribuție reconfigurabil, ținând cont de restricția de radialitate și de restricțiile operaționale. Comparația este realizată utilizând o variantă propusă a unui indicator bazat pe dominanța stocastică de ordinul întâi.*

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Cuvinte cheie: sisteme electrice de distribuție, problemă de reconfigurare, pierderi minime de putere, îmbunătățire progresivă, călire simulată, dominanță stocastică.

1. Introduction

This paper deals with the distribution system reconfiguration problem. Distribution systems are characterized by weakly meshed structure and radial operation. Hence, new configurations for the electrical network can be obtained by opening and closing different branches.

Theoretically, for a specified objective function (i.e., minimization of power losses, minimization of voltage deviations, minimization of branches loading level, and others), a globally optimal solution would be determined after analyzing all possible radial configurations of the network. However, the analysis of all possible radial configurations becomes impossible for large distribution networks due to the enormous and practically intractable calculation time [1]. In this case, deterministic and meta-heuristic methods are used to solve this problem [2]. For the generation of radial configurations, these methods are based on the branch-exchange mechanism [3], which involves the formation of a loop by closing an open branch and returning to a radial configuration by opening another branch from the formed loop.

Two methods for distribution system reconfiguration are considered in this paper, namely, the iterative improvement algorithm as a representative of deterministic methods, and the simulated annealing algorithm as a representative of meta-heuristic probability-based methods [4]. For the comparisons among the results, it is necessary to resort to suitable metrics. Many literature references calculate only the best, average and worst solution found. However, these calculations do not provide a global view on the solutions obtained by the methods. In general, given the extremely high number of possible configurations for real-scale distribution systems, there is no guarantee that the global optimum may be reached. For the meta-heuristic methods, the variability of the solutions depending on the random numbers involved in the probability-based calculations require at least the execution of a statistically significant number of solutions to extract information on the characteristics of the algorithm applied to the specific problem [5]. This paper calculates a global metric based on the first-order stochastic dominance.

The specific contributions of this paper are:

1. The discussion and testing of the deterministic optimization method by changing the order with which the list of the open branches is

visited, leading to possible different solutions due to the local optima found by the deterministic method.

2. The proposal of a variant of the indicator [6] based on the first-order stochastic dominance, in such a way to link the results with each other in a relative way.

The next sections of this paper are organized as follows. The mathematical model of this problem is presented in Section 2. Section 3 and Section 4 provide information about the implementation of the selected deterministic and meta-heuristic solution methods, respectively. Section 5 presents the stochastic dominance-based comparison method with the proposed variant. A numerical example for a test distribution network is presented in Section 6 and the conclusions are reported in Section 7.

2. Problem formulation

In this paper, the primary aim of the reconfiguration process is the minimization of the power losses. In this paper, the simple optimization with a single set of load values (single snapshot) is considered, without loss of generality, as the aim of the paper is to show the mechanism of comparison among different sets of solutions. In more general cases, also considering the presence of distributed generation [7], the objective function may be formulated in a time period partitioned into time steps [8], or as a multi-objective function [9].

For a distribution system with N_k nodes (slack node excluded) and N_b branches, in which R_i is the branch resistance and I_i is the branch current magnitude at branch $i = 1\ldots, N_b$, the optimization problem is expressed as follows:

$$\min f(X) = \sum_{i=1}^{N_b} R_i I_i^2 \tag{1}$$

The optimization problem is subject to the following constraints:

- Voltage magnitude constraints:

$$V_i - V_i^{max} \leq 0, \quad i = 1 \ldots N_k \tag{2}$$

$$V_i^{min} - V_i \leq 0, \quad i = 1 \ldots N_k \tag{3}$$

- Thermal limit constraint

$$I_i - I_i^{max} \leq 0, \quad i = 1 \ldots N_b \tag{4}$$

- Network radiality constraint, handled with the branch exchange mechanism, so that only radial solutions are generated in the solution process.

The branch exchange mechanism is based on the following steps:

- Selection of a branch from open branches list.
- Closing the selected branch to form a loop.
- Selection of a branch to open from the formed loop to obtain a new radial configuration.

3. Iterative Improvement Algorithm

The iterative improvement algorithm is a deterministic method based on the repeated application of the branch exchange mechanism. Starting from the initial configuration represented by the ordered list of open branches, the algorithm searches for new configurations with a lower objective function by making successive topology transformations. The objective function is evaluated through a power flow calculation, using the backward-forward sweep algorithm.

The steps of the iterative improvement algorithm are as follows:

- Save the objective function value for the initial configuration of the network.
- Make a list with all the open branches of the network.
- Establish a closing order for the open branches from the defined list.
- Close the branches sequentially, one at a time.
- For each closed branch, all the branches in the formed loop are opened sequentially, and for each case the power flow calculation is performed.
- Save the configuration with the minimum power losses and update the network topology.
- Update the list with all the open branches of the network.
- Repeat the process until the list with open branches remains unchanged between two successive iterations.

An advantage of this method is the high speed of finding an acceptable solution. The major drawback is represented by the fact that this algorithm can easily stop in a local minimum solution.

The solutions found with the iterative improvement algorithm depend on the order with which the list of initial open branches is visited. Because of that, different solutions may arise from the deterministic algorithm, none of which can be seen as a possible global optimum, because any solution of the iterative improvement algorithm may fall into a local optimum. Since the

iterative improvement algorithm is relatively fast, a plurality of solutions may be handled by changing the order of the branches in the list of the initial open branches. For a distribution system in which the list of open branches contains N_o open branches, there are $N_o!$ possible permutations of the order with which the open branches can be listed. For a real-size network, this number could be excessively high to determine all the permutations. In addition, these permutations refer to only one radial configuration of the initial network (the one taken as the reference). It is then clear that the repeated use of the iterative improvement algorithm has limitations to find the best solutions for the optimal reconfiguration problem.

4. Simulated Annealing Algorithm

The simulated annealing algorithm is a meta-heuristic method inspired from the annealing process in which a metal is heated up to a maximum value at which all particles of the solid randomly arrange themselves in the liquid phase and then is cooled slowly enough until the particles arrange themselves in a ground state of a solid [10]. The ground state of the solid means a state where the electrons are in their lowest possible energy configuration within the atom and for the algorithm means a global optimum solution.

The algorithm starts from an initial solution and generates, at each iteration, a random neighbor which represents another possible solution for the optimization problem. After generating the neighbor, the following cases are possible depending on the value of the objective function:

- If the objective function is improved, the neighbor is accepted.
- If the objective function is worsened, the neighbor is subject to a specific test with a given probability; if the test is passed, the neighbor is accepted, otherwise it is rejected.

The possible selection of a neighbor solution that worsens the value of the objective function depends on the current value of the control variable (the "temperature" T) and the amount of degradation ΔF of the objective function. The testing condition is based on the following value:

$$P(\Delta F, T) = e^{-\frac{f(x\prime)-f(x)}{T}} \tag{5}$$

with:

$f(x')$ – objective function value for the generated neighbor solution x'

$f(x)$ – objective function value for the current best solution x

ΔF – degradation of the objective function, $\Delta F = f(x') - f(x)$
T – temperature value

The algorithm is composed of two evolution cycles:
- External cycle
- Internal cycle

The external cycle is responsible for cooling process control. In the proposed algorithm, the temperature T_k at iteration k is updated in a geometric way [11], where α is the cooling rate, $\alpha \in (0,1)$:

$$T_k = \alpha T_{k-1} \tag{6}$$

The internal cycle is responsible for generating and evaluating new solutions. In this cycle, the following steps are taken:

- A random open branch is selected to be closed, forming a loop.
- A random branch from the formed loop is selected to be opened.
- The new configuration is evaluated based on the power flow calculation.
- The new configuration is accepted if the objective function for the generated neighbor is lower than the objective function for the current solution. If the objective function for the generated neighbor is higher than the objective function for the current solution, the new configuration is accepted with a probability, according to Equation (5). If the value $P(\Delta F, T)$ calculated for the testing condition is greater than a random generated number $r \in [0,1]$ the solution is accepted, otherwise the solution is rejected.

The above steps of the internal cycle are repeated until the stop criterion is met. The stopping criterion for the internal cycle is represented by the simultaneous application of two conditions:

- reaching the maximum number of accepted configurations M_C (active for high values of the temperature T, when the iterative process is far from the final solution, and it is not necessary to have too many iterations of the internal cycle);
- reaching the maximum number of analyzed configurations M_A (active for low values of the temperature T, as the number of accepted solutions becomes lower and lower, until no solution leading to a true improvement is found).

5. Comparisons among the solution methods

Since the methods considered are parts of different categories (deterministic and meta-heuristic) and random numbers are involved in meta-heuristic methods, it is necessary to use a comparison metric that considers specific information about the probabilistic distribution of the solutions [12].

In [6] is proposed a new performance indicator based on the concept of first-order stochastic dominance that considers the probabilistic distributions of the solutions found with different algorithms for the problems where the global optimum is not known. In this work, the cumulative distribution function (CDF) profiles are used to compare the results.

Considering a single-objective optimization problem, with a generic number *M* of solving algorithms and *H* executions for each algorithm, the empirical CDFs are constructed sorting in the ascending order the *H* solutions obtained for the objective function for each algorithm.

To compare the obtained CDFs, a reference CDF is necessary. To construct this reference CDF, the results from all the methods are merged and sorted in the ascending order into a temporary vector. From this vector, only the first *H* elements are kept. In this way, the reference CDF contains the lowest values of the objective function and satisfies the first-order stochastic dominance conditions. In particular, the reference CDF is characterized by the fact that there are no values located on the left of it.

After determining the reference CDF, the next step is to calculate the areas between each CDF for the algorithms $m = 1, \dots, M$ and the reference CDF, based on the same number H of points:

$$A_m^H = \frac{1}{H} \sum_{x=1}^{H} \left(f_m^H(x) - f_{\text{ref}}^H(x) \right) \tag{7}$$

After the calculation of these areas, the last step applied in [6] is to calculate a performance indicator, called *OPISD (Optimization Performance Indicator based on First-order Stochastic Dominance)*:

$$OPISD_m^H = \frac{1}{1 + A_m^H} \tag{8}$$

Note that the maximum value for this indicator is equal to unity, when $A_m^H = 0$ as a result of a total contribution of the solutions from the tested method to the formation of the reference CDF.

The *OPISD* indicator is fully viable to obtain a ranking of the result from different algorithms represented by the CDFs of their solutions. However, the numerical values of the *OPISD* indicator depend on the order of magnitude of the quantity represented on the horizontal axis of the CDFs. A specific variant is proposed in this paper, in which the area is determined in a relative way, taking as a *worst ideal area* $\tilde{A}^H$ the area determined by taking the difference between the best and the worst solution found in all the calculations, which is equal to the difference itself, as the length of the vertical coordinate is equal to unity by definition of CDF:

$$\tilde{A}^H = \max_{\substack{x=1,\dots,H \\ m=1,\dots,M}} \{f_m^H(x)\} - \min_{\substack{x=1,\dots,H \\ m=1,\dots,M}} \{f_m^H(x)\} \tag{9}$$

Based on the above formulation, the variant considered for the *OPISD* indicator is defined as follows:

$$\widetilde{OPISD}_m^H = \frac{1}{1 + \frac{A_m^H}{\tilde{A}^H}} \tag{10}$$

To include the limit case in which all methods find the same solution and $\tilde{A}^H = 0$, for the case $\tilde{A}^H = 0$ it is imposed that $\widetilde{OPISD}_m^H = 1$.

6. Numerical example

The proposed algorithms were tested on the IEEE 118 bus test system (Figure 1). This system is a 11 kV distribution network that contains 118 normally closed branches and N_o = 15 redundant branches (119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133). The input data for lines and loads are taken from [13]. The system load is assumed to be constant, the total demanded power being 22709.72 kW and 17041.07 kvar. In this configuration, the initial power losses are 1296.57 kW and the minimum node voltage of the system is 0.8688 pu, which occurs at bus 77.

The base values considered are $V_{\text{base}} = 11$ kV and $S_{\text{base}} = 100$ MVA and the substation voltage is $V_0 = 1$ pu.

For this network, the total number of possible radial configurations is about $3.853 * 10^{15}$. Using an Intel Core i5-8300H @ 2.30 GHz computer, the estimated time to analyze all these configurations would be of the order of

several millennia. In addition, for N_o = 15 open branches, the number of permutations for analyzing a single list of open branches is as high as $N_o! = 1.31 * 10^{12}$.

In order to perform a statistical analysis of the results, H = 500 runs were performed for each algorithm.

The best results obtained for this network are presented in Table 1 and Figure 2. The values of the simulation parameters used in the simulated annealing algorithm were $T_0 = 2.1$, $M_A = 400$, $M_C = 100$, and $N_{s,max} = 25$.

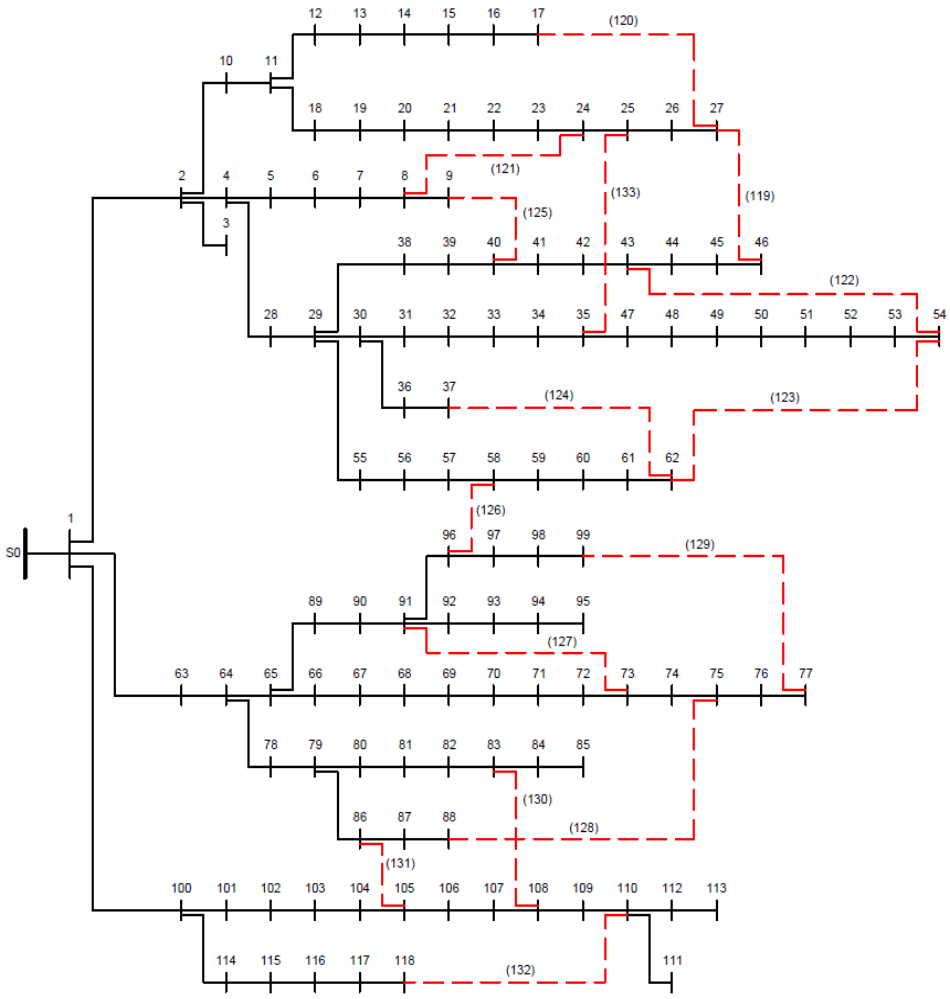

Figure 1. IEEE 118 distribution system (initial configuration).

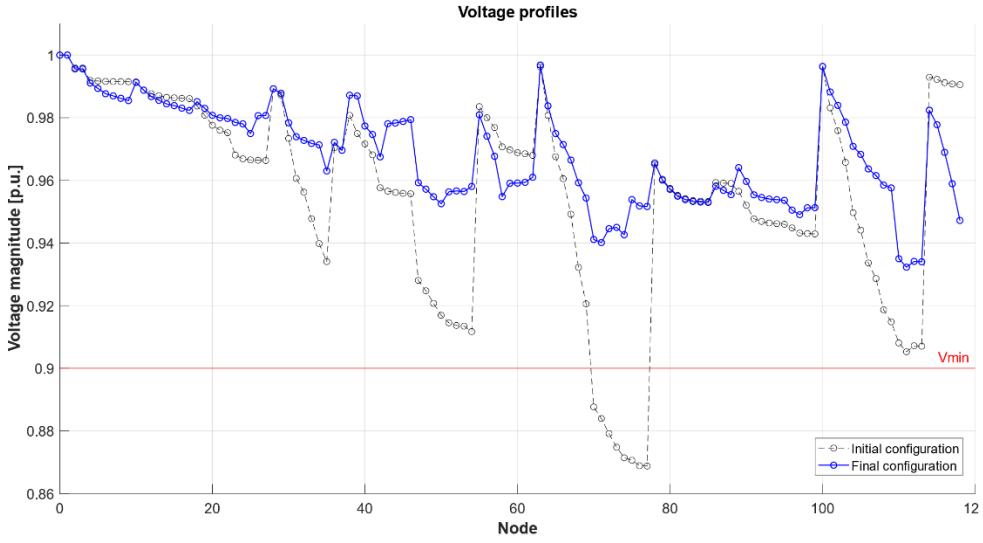

Figure 2. Voltage profiles for initial and final configurations (IEEE 118 network).

Analyzing the results, it can be seen that after the reconfiguration process, a reduction of 34.2% was obtained for the power losses.

Table 1. **The best obtained results for IEEE 118 network**

	Before reconfiguration	**After reconfiguration**
Open branches	118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132	24, 26, 35, 40, 43, 51, 59, 72, 75, 96, 98, 110, 122, 130, 131
Power losses (kW)	1296.57	853.58
Minimum voltage (pu)	0.8688	0.9323
Mean CPU time/run (s)	0.78	47.04

Also, it is observed that the voltage level at nodes 73, 74, 75, 76, 77, 78, 79 and 80 is below the minimum limit in the initial configuration. After the reconfiguration process, the voltage profile has improved, the minimum voltage increasing from 0.8688 pu to 0.9323 pu.

There are several papers in the literature that analyze this network, such as [14], [15], [16], but most of them do not specify the used base values and for this reason the results are similar, but not identical (i.e., initial power losses). However, the results obtained have been validated by those in [17].

The final configuration of the network is presented in Figure 3.

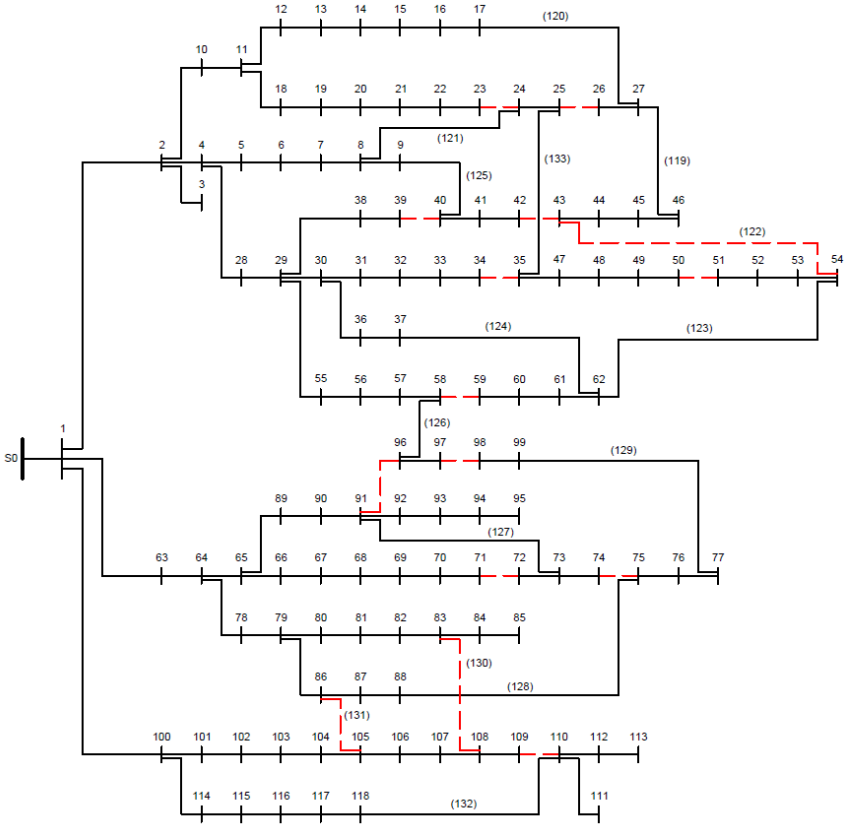

Figure 3. IEEE 118 distribution system (final configuration).

To compare the solution methods $m = 1, \ldots, M$ and to determine which one is performing better, the differences between the results should be quantified using CDF curves and $\widetilde{OPISD}_m^H$ indicators. The CDFs for both algorithms are presented in Figure 4 and Table 2. The worst ideal area with $H = 500$ is $\tilde{A}^H = 9.2965 * 10^{-5}$.

Note that for the cooling rate parameter three different values were considered: $\alpha_1 = 0.9$, $\alpha_2 = 0.7$ and $\alpha_3 = 0.5$.

By analyzing Figure 4, it can be observed that the meta-heuristic method reached the minimum power losses in proportion of 88% for $\alpha_1 = 0.9$, compared to deterministic method who reached the minimum power losses in proportion of only 26%. Also, for $\alpha_2 = 0.7$, the meta-heuristic

method reached the minimum power losses in proportion of 72% and in proportion of 61% for $\alpha_3 = 0.5$.

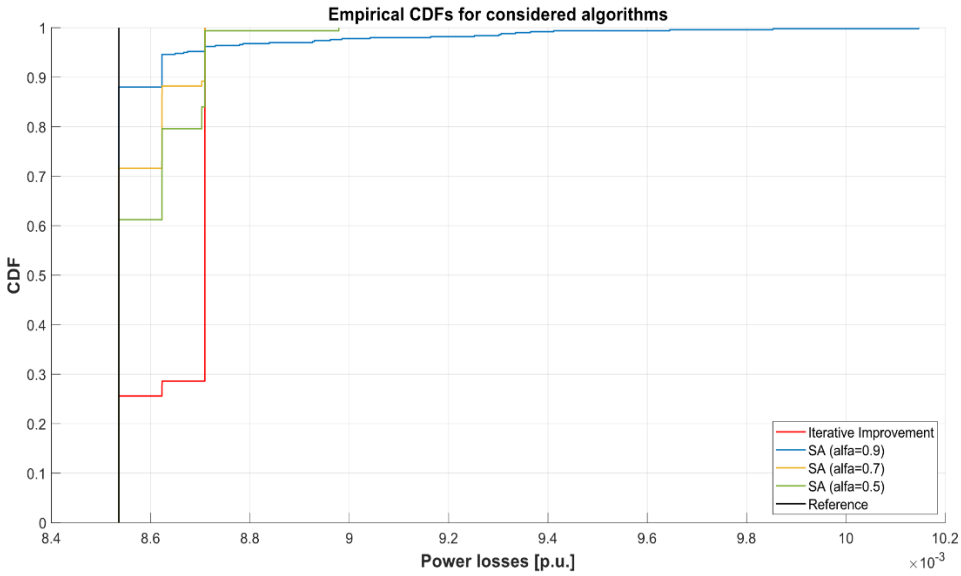

Figure 4. CDFs for considered algorithms (IEEE 118 network)

Table 5.4. **$\widehat{OPISD}$ for the algorithms applied to the IEEE 118 network (H = 500)**

Algorithm *m*	**Area A_m^H**	**$\widehat{OPISD}_m^H$**	**Ranking**
Simulated Annealing $\alpha_1 = 0.9$	$3.3376 * 10^{-5}$	0.7358	1
Simulated Annealing $\alpha_2 = 0.7$	$3.4802 * 10^{-5}$	0.7276	2
Simulated Annealing $\alpha_3 = 0.5$	$5.269 * 10^{-5}$	0.6383	3
Iterative Improvement	$1.2634 * 10^{-4}$	0.4239	4

Regarding the deterministic method, it can be seen that the algorithm reaches a few different solutions and is stuck in most of the cases into a local minimum solution different from the relative best solution obtained by the same deterministic method. Also, the simulated annealing algorithm finds a few relatively good solutions, followed in one case by many worse solutions, the worst case of which determines the worst ideal area.

7. Conclusions

This paper presents a comparison between two different solution methods for solving the reconfiguration of electrical distribution systems considering the minimum power losses as the general objective. The comparison is carried out by considering a proposed variant of the first-order stochastic dominance indicator defined in the literature.

Analyzing the results, it can be seen that the simulated annealing algorithm provides solutions with lower total losses in most of the cases, compared with the iterative improvement algorithm. It is important to mention that the meta-heuristic algorithm requires a very high computation time, compared to the deterministic algorithm, especially for large networks.

In this paper, the behavior of the simulated annealing has been analyzed by changing the cooling rate value. As expected, for the considered network, a high value for this parameter implies a more accentuated component of exploring the space of solutions, thus obtaining better solutions.

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