THE HYDROCARBONS DISPLACEMENT BY WATER IN A NON-UNIFORM RESERVOIR

DISLOCUIREA HIDROCARBURILOR DE CĂTRE APĂ ÎNTR-UN ZĂCĂMÂNT NEUNIFORM

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Abstract: *The paper treats the problem of water injection into subsaturated reservoirs to describe their evolution in time. The reservoir, is considered a continuously stratified system with different rates of uniformity. The obtained results had show that in the case of a uniform reservoir there is a period in which, practically, it is produced only gas, but a high rates of uniformity and high viscosity of oil, the resaturating process continues in the oil-water production period. It shows too, that in case of the increased number of layers, the displacement with the mixture of the oil-water zone is possible to be approximated with the piston-like one.*

Keywords: reservoir, linear system, algorithm, waterfront, oil-gas mixture, displacement process, initial saturation.

Rezumat*: Lucrarea tratează problema injectării apei în rezervoare subsaturate pentru a descrie evoluția acestora în timp. Zăcământântul, este considerat un sistem stratificat continuu cu rate diferite de uniformitate. Rezultatele obținute arată că în cazul unui zăcământ uniform există o perioadă în care, practic, se produce doar gaz, dar la o rată mare de uniformitate și vâscozitate ridicată a țițeiului, procesul de resaturare continuă în perioadă producției țiței-apă. De asemenea, se arată că în cazul numărului crescut de strate, dislocuirea zonei amestecului țiței-apă poate este posibilă să fie aproximată cu cea de tip piston.*

Cuvinte cheie: *rezervor, sistem linear, algoritm, front de apă, amestec țiței-gaze, proces de dislocuire, saturații inițiale.*

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1. Introduction

In a linear (non-uniformly vertical) system, Dykstra and Parsons [1] described the problem of the displacement of crude oil by water, considering that the system consists of a series of continuously separated layers that communicate with each other.

Thus the problem is reduced to the overlapping over time of the effects of displacement in each layer.

It is considered that behind the waterfront, the saturation in crude oil is instantly reduced to the value of the irreducible saturation in crude oil, a hypothesis that can be made at a very large number of layers. The problem was also treated by Muskat,[2] thus considering an exponential variation of the permeability on the layer thickness so that the reservoir is uniform horizontally.

This research addresses the issue of the displacement of a mixture of crude oil and gas by water, in the same model of the continuously stratified reservoir. A calculation algorithm has been developed regarding the overlapping overtime of the displacement effects in each component layer of the system. To describe the process of displacement of the crude oil-gas mixture by water on each component layer (layer considered uniform), a mathematical model was used [3], for the case of piston-type displacement of crude oil by water.

The results obtained led to the conclusion that, compared to the uniform reservoir (in which the amount of crude oil extracted during the resaturation process represents a negligible amount compared to the previously extracted one), in the case of the non-uniform reservoir (vertical) the amount of crushed oil can reach significant percentages of the amount extracted by the end of the operation. Also, several working variants were performed for different rations of non-uniformity and viscosity of crude oil.

2. Definition and concepts

It is considered the same model as the reservoir Dykstra and Parsons, the reservoir is linear, L – length and B - width stratified continuously. The total thickness can be written like this:

$$H = \sum_{i=1}^{n} h_i \tag{1}$$

where: h_i – represents the thickness of the component layer i;

H – represents the total layer thickness.

The component layers have the same porosity *m*, being initially saturated with the same fluids: crude oil, gases, and interstitial water saturation s_{t0}, s_{g0}, and s_{aint}.

The same curves of relative phase permeability are characteristic of all layers. Thus, in this case, each layer will be characterized by the same irreducible saturation in crude oil $s_{ţir}$ and the same irreducible saturation in gases s_{aid}.

The system is operated at constant pressure drop (ΔP) at extraction and injection. The respective layers are distinguished by the absolute permeability's K_i characteristic of each layer *I* (*fig. 1*).

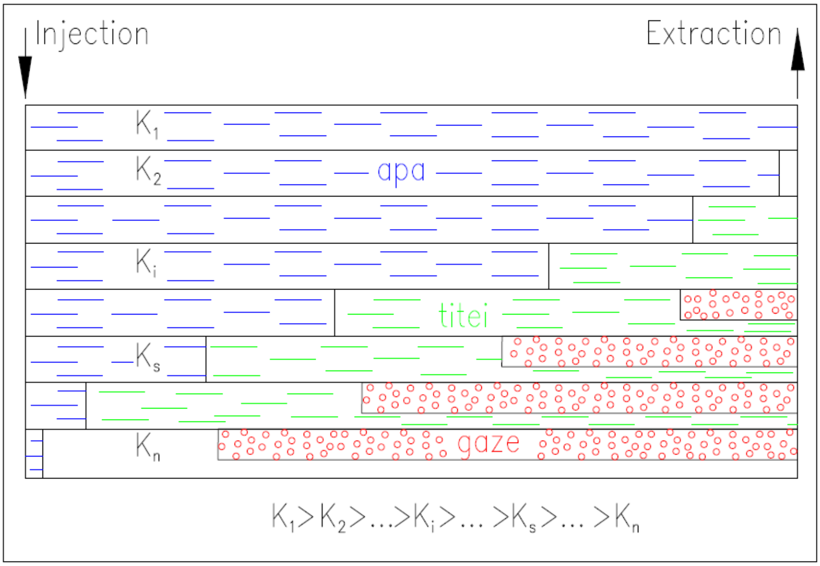

Fig. 1. Schematic representation of the permeability of the layers.

Given a uniform linear *j* – layer (identical to one of the components of the system shown above) [4], in which the gas front is at a distance x_0 from the extraction well, the time of gas depletion at the injection of the displacement agent (water) of the crude oil – gas mixture, is defined by equation (2):

$$\Delta t_g = \frac{m}{K_j c_g}\left[2B_g\left(\frac{x_g}{L}\right)_j - A_g\left(\frac{x_g}{L}\right)^2\right] \qquad (2)$$

where: *m* – represents the porosity;

K_j – represents the absolute permeability for component *j*.

where: $$A_g = A_g - M_a M_t M_g \quad (3)$$

$$B_g = \left(M_t^* + M_g\right) M_a M_g + \frac{\Delta S_g}{\Delta S_a}\left(M_t^* + M_g\right)^2 (M_t - M_a) \quad (4)$$

$$C_g = \frac{2\Delta P}{L^2 - \Delta S_g} M_a M_t M_g^2 \quad (5)$$

$$\Delta S_a = 1 - S_{a\,int} - S_{tir} - S_{gir} = \Delta S_t \quad (6)$$

$$\Delta S_g = S_{g0} - S_{gir} \quad (7)$$

$$M_t^* = \frac{k_t^*}{\mu_t} \quad (8)$$

where: M_t^* – represents the oil mobility in the area of the oil-gas mixture.

It can be written as the total time of gas depletion on a layer *j*, (duration of the re-saturation process on layer *j*), as follows from the equation (2) for $x_g/L = 1$:

$$T_{ep*g*j} = \frac{m}{K_j C_g}\left(2B_g - A_g\right) \quad (9)$$

At the time of gas depletion, this crude oil-water front is in the position:

$$x_{a*ep*g} = L\frac{\Delta S_g}{\Delta S_a} * \frac{M_i + M_a}{M_g} \quad (10)$$

Therefore, the time required for the complete flooding of the layer is:

$$\Delta t_a = \frac{mL^2 \Delta S_a}{K_j \Delta P M_t}\left\{\frac{1}{2}\left(\frac{M_t}{M_a} - 1\right)\left[1 - \left(\frac{x_a}{L}\right)^2\right] + \left(1 - \frac{x_a}{L}\right)\right\} \quad (11)$$

According to equations (9) and (11), the gases will be exhausted for the time from the more permeable layers, just as they will be the ones that will be flooded first. However, until the depletion of gas in a single layer *j*, the position of the crude oil front at time *T* from the start of the injection is given by the equation:

$$\left(\frac{x_a}{L}\right)_j = \frac{B_g - \sqrt{B_g^2 - A_g D_a}}{A_g} \quad (12)$$

where: $$D_g = -A_g + 2B_g \frac{K_j}{m} C_g T \quad (13)$$

The position of the crude oil-water front on the same layer is given by the equation:

$$\left(\frac{x_a}{L}\right)_, = \frac{-B_a+\sqrt{B_a^2+\frac{K_j}{m}-A_aC_aT}}{A_a} \tag{14}$$

where:

$$A_a = \frac{\Delta S_a}{\Delta S_g} A_g \tag{15}$$

$$B_a = \left(M_t^* - M_g\right)M_aM_t \tag{16}$$

$$C_a = \frac{2\Delta P B_a}{L^2 \Delta S_a}\left(M_t^* + M_g\right) \tag{17}$$

The injection flow at time T on layer j is:

$$Q_{inj} = \frac{Bh_jK_j\Delta P}{\frac{x_a}{M_a}+\frac{L-x_a-x_g}{M_t}+\frac{x_0}{(M_t^*+M_g)}} \tag{18}$$

The flow extracted from crude oil and gas is:

$$Q_{t\,j}^* = Q_{inj\,j}\frac{M_t^*}{M_t^*+M_g} \tag{19}$$

Between two successive moments, T_i and T_{i+1} from the layer j from which the gases have not been exhausted, several gases will be extracted:

$$\Delta\left(\Delta G_j\right)_i^{i+1} = V_{pj} * \Delta S_g\left[\left(\frac{x_g}{L}\right)_i - \left(\frac{x_g}{L}\right)_{i+1}\right] \tag{20}$$

Where: $V_{pj} = m*B*h_j*L$ and represents the pore volume of layer j.

The amount of water injected into this layer during the same period is:

$$\Delta\left(\Delta W_j\right)_i^{i+1} = V_{pj} * \Delta S_a\left[\left(\frac{x_g}{L}\right)_{i+1} - \left(\frac{x_g}{L}\right)_i\right] \tag{21}$$

where: W – represents the injected water.

Given that layer j has not yet been flooded, the amount of water extracted between the two successive moments is:

$$\Delta\left(\Delta\omega_j\right)_i^{i+1} = 0 \tag{22}$$

where: ω – represents the extracted water.

From layer j was extracted from T_i la T_{i+1} a quantity of crude oil:

$$\Delta\left(\Delta N_j\right)_i^{i+1} = \Delta\left(\Delta W_j\right)_i^{i+1} - \Delta\left(\Delta G_j\right)_i^{i+1} \tag{23}$$

On another layer *l*, from which the gases have been depleted, the position of the crude oil – waterfront at some point T_{i+1} is:

$$\frac{x_a}{L} = \frac{-M_a + \sqrt{M_a^2 + (M_t - M_a) * C_n}}{M_t - M_a} \tag{24}$$

where:

$$C_n = (M_t - M_a) * \left(\frac{x_a}{L}\right)_i^2 + 2 * M_a * \left(\frac{x_a}{L}\right)_i + \frac{2 * K_l * \Delta P}{L^2 * m * \Delta S_a} * M_a * M_t * (T_{i+1} - T_i) \tag{25}$$

$\left(\frac{x_a}{L}\right)_i$ – represents the position of the crude oil-water front on layer *l* at time T_i.

At the time T_{i+1} a flow of water is injected on layer *l*:

$$Q_{inj\ l} = \frac{B * h_l * K_l * \Delta P}{L * \left[\frac{\frac{x_a}{L}}{M_b} + \frac{1 - \frac{x_a}{L}}{M_t}\right]} \tag{26}$$

There is crude oil flow: $Q_t = Q_{inj}$ (27)

Given the fact that: $Q_a = 0$ and $Q_{inj} = 0$, between the two successive moments of T_i and T_{i+1} an amount of water is injected into the layer:

$$\Delta(\Delta W_l)_i^{i+1} = V_{pl} * \Delta S_a \left[\left(\frac{x_a}{L}\right)_{i+1} - \left(\frac{x_g}{L}\right)_i\right] \tag{28}$$

A quantity of crude oil is produced:

$$\Delta(\Delta N_l)_i^{i+1} = \Delta(\Delta W_l)_i^{i+1} \tag{29}$$

Because: $\Delta(\Delta G_l)_i^{i+1} = 0$ and $\Delta(\Delta \omega_l)_i^{i+1} = 0$ (30)

According to the previous scenarios on a layer where the waterfront has reached the extraction area, water will be produced at a flow rate equal to the flow of water injected on that layer as follows:

$$Q_{a\ j} = Q_{inj\ j} = \frac{B * h_j * K_j * \Delta P * M_a}{L} \tag{31}$$

Assuming that we are at some point during the operation, characterized by the fact that the most permeable layers were flooded, on the next layers the gases were depleted, but the crude oil – waterfront has not yet reached the extraction area, and from the layers, with low permeability, the gases were not depleted either [5].

On the other hand, given the scenarios made at the beginning of the research, the behavior of the system as a whole will not depend on the relative position of the layers in the system. This allowed the fictitious arrangement of the layers in a convenient order, namely their interpretation was made in descending order of absolute permeability: $K_j \geq K_l$ for $l \geq j$.

If two states have the same absolute permeability (given their behavior), a single layer equivalent to the thickness equal to the sum of the thicknesses of the two layers was considered.

3. Results and discussions

It is considered the system consisting of *n* layers. The indicated situation is reached (figure 1), which allowed the elaboration of the following algorithm:

- calculate with equation (9) the time T_1 in which the gases in layer 1are exhausted (most permeable). Correspondingly calculate with equations (12) and (13) the positions of the gas and crude oil fronts on all other layers at time T_1, then apply in the algorithm the equations (14 – 19) determining the operating parameters on each layer;
- the summation is performed to obtain the exploitation parameters of the entire reservoir;
- is calculated with equation (1) the time required further (Δt_{g1}) for the depletion of the gases in the second layer;
- is calculated further time required (Δt_{a1}) for flooding of layer 1;
- if $\Delta t_{g1} < \Delta t_{a1}$ it is calculated on each component layer; the operating parameters being summed over time $T_2 = T_1 = \Delta t_{g1}$;
- the calculations are continued until $\Delta t_{g1} > \Delta t_{a1}$ then the exploitation parameters are calculated on each component layer realizing the sum at $T_l = T_{l-1} + \Delta t_{g1}$ (at the moment when layer 1 is flooded, and on layer *l* the gases have not been exhausted). At this point layer *l* will produce only water, layer 2 to layer (*l-1*) will produce only crude oil, and layer *l* to layer *n* will produce both crude oil and gases;
- the operating parameters are calculated on component layers and by summing up the whole system at the respective gases depletion times on layers' $l - k$, when layer 2 has not been flooded (*fig. 2*) [6];
- the calculations are continued until the flooding of layer *m* but on layer *j* the gases have not yet been exhausted ($j \geq k$);
- the calculations are continued until the depletion of gases in layer *n* but in layer *s* the gases have not yet been flooded ($s \geq m$);
- the calculations are continued until the flooding of layer *n*.

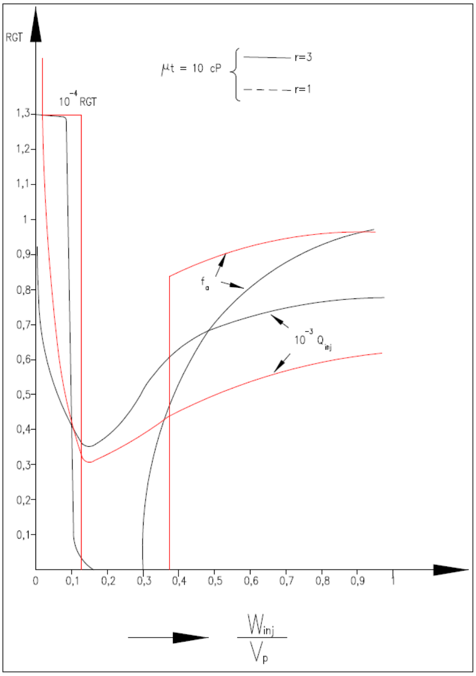

Fig. 2. Ratio variation W_{inj}/V_p depending on RGT, injection flow, and water fraction.

Based on this algorithm, the previously presented calculations were simulated with specialized software. The simulation was applied to calculate the process of displacement of a crude oil-water mixture by water in a continuous stratified system consisting of about 50 layers.

It started from the presume according to which the layers have a thickness equal to 30 cm. The length of the deposit is about 1470 m, the width is 3450 m, and the total effective thickness is 15 m. The component layers were allowed to be uniform with a porosity of 0.26 being initially saturated with the same fluids: crude oil, gas, and interstitial water.

The initial saturations in these fluids are: $S_{t0} = 0{,}61$; $S_{t0} = 0{,}14$ and $S_{a\ int} = 0{,}25$. Residual saturation in crude oil is considered to be $S_{tir} = 0{,}27$ and the residual gas saturation is: $S_{gir} = 0$.

Also, the value of the volume factor for gases on the considered reservoir is $b_g = 0{,}0075$, and the value of the crude oil volume factor is $b_t = 1{,}14$.

The viscosity of the gas is 0,01 cP, and the viscosity of the water is 1 cP.

The process of displacement of the oil-gas mixture by the injected water takes place at a constant pressure drop or a variable injection flow: $\Delta P = 50\ at$.

Given that the displacement process is of the piston type, behind the crude oil – waterfront relative permeability of the water will be $k_a = 0{,}23$. In the area saturated only with crude oil (the area between the water – crude oil front and the crude oil – gas front), the permeability for crude oil will have a constant value: $k_ț = 0{,}28$, and in front of the crude oil – gas front in the crude oil – gas mixing zone (given that the gas – crude oil saturations in this area are constant during the displacement process), the relative phase permeabilities will be constant: $k_ţ = 0{,}48$ şi $k_g = 0{,}04$.

The average absolute permeability of the reservoir is 210 mD. This absolute permeability y is calculated as a weighted average based on the thickness. The distribution of absolute permeability's over the entire thickness of the reservoir was considered exponential, if the absolute permeability's characteristics of each component layer of the system are in order and the absolute permeability of each component layer is represented in a semi-logarithmic graph as a function of the cumulative thickness, a straight line will be obtained.

The ratio between the maximum and the minimum value of the absolute permeability (non-uniform ratio) can be characterized in this case by the non-uniformity of the reservoir.

Six non-uniformity reports were selected for which such calculations were performed: $r = 3; 5; 10; 20; 50; 100$, in each case keeping the value of 210 for the absolute average permeability of the whole system. Also for each ratio of non-uniformity, six working variants were made for the viscosities of crude oil: $\mu_t = 2; 5; 10; 20; 50; 100$ cP.

In (*fig. 2 – 4*) shows some examples of variation in the exploitation parameters of a continuously stratified non-uniformity reservoir. In (figure 2) the variation is identified on an almost uniform reservoir, since $r = 3$, *RGT* (RGT – oil gas ratio), the fraction of water f_a in the stream, and the injection flow in case the viscosity of crude oil is 10 cP.

The comparison shows the variation of *RGT*, f_a, and Q_{inj} on the same reservoir but is considered uniform ($r = 1$), and the existence of a crude oil-water mixing zone behind the waterfront was taken into account.

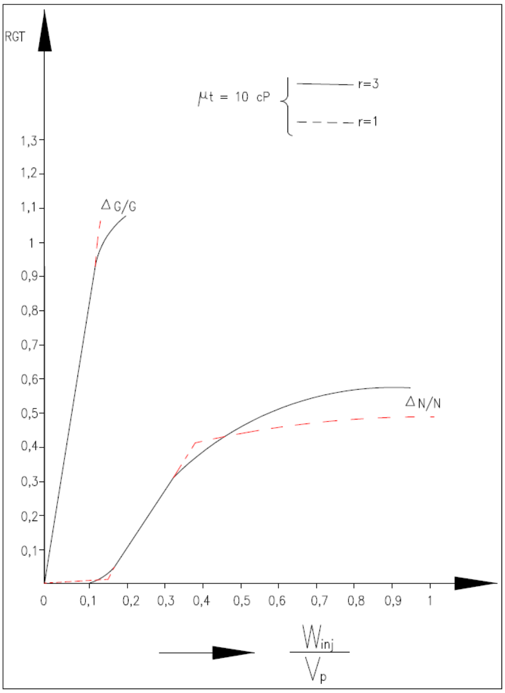

Fig. 3. Ratio variation W_{inj}/V_p depending on RGT, ΔG/G, and ΔN/N.

As can be seen from a large number of layers, the hypothesis made regarding the instantaneous establishment of a residual saturation in crude oil behind the waterfront (displacement of piston–type crude oil) does not affect too much the prediction of the evolution of the exploitation parameters, the prediction of their real evolution, but as it results from (figure 3) and as it was expected otherwise the prediction in this hypothesis of the final recovery factor (for which the value obtained at ($f_a = 0,95$) is more optimistic.

In the cases shown in (*fig. 2 and 3*) (due to the small degree of non-uniformity at $r = 3$ the reservoir can be considered as uniform), the depletion of gases in the least permeable layer was done long before the appearance of water on the most permeable layer, which has not happened in the case of systems with a high degree of non-uniformity (*table 1 and fig. 5*). In the case of these reservoirs, at the time of depletion of gases in one component layer, the flow of crude oil as a whole but at the time of the flooding of another component layer, the flow of crude oil as a whole degree.

Table 1. **The variation ΔG/G (%) at the time of water appearance in the extraction area**

$r = \frac{K_{max}}{K_{min}}$	***The oil viscosities (μ_t, cP)***					
	2	***5***	***10***	***20***	***50***	***100***
3	100	100	100	100	100	100
5	100	100	100	100	100	100
10	99,95	99,9	99,8	99,7	99,3	99,1
20	95,3	95	94,4	94,3	93	92,2
50	85,8	85,3	84,5	83,4	82,2	81,1
100	78	77,7	76,8	75,6	74,3	73,1

On the other hand, the flowing water extracted from the flooded layers has a constant value, which is why during the operation it causes some fluctuations of the ordinal of 1 – 3 % of the current water fraction.

In (*fig. 4*) is shown the variation of the exploitation parameters on such a reservoir ($r = 100$, $\mu_t = 100$ cP) but the f_a curve is linear.

It can be seen from this figure that, although they were not depleted from the system ($\Delta G/G = 0,998$), the fraction of water in the stream reached a value of 95 %, and the recovery factor reached the value of 36,5 %.

The idea is repeated in (*table 2 and fig.* 6) where it is observed that if you go with the exploitation of non-uniform stratified reservoirs continuously until the complete depletion of gases (even from the least permeable layer which means the end of the oil re-saturation period), important recovery factors for crude oil are reached. The variation of this recovery factor obtained at the end of the oil re-saturation period for different degrees of non-uniformity and containing crude oil of different viscosities is shown.

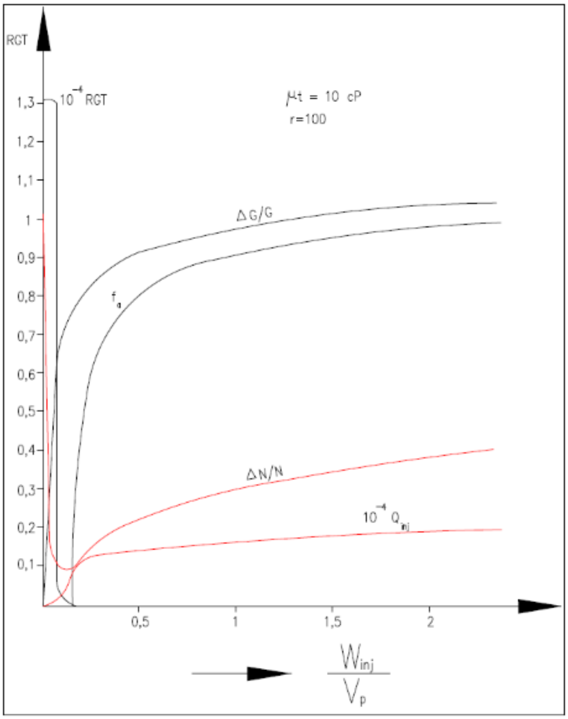

Fig. 4 – Ratio variation W_{inj}/V_p depending on RGT, ΔG/G, ΔN/, N and injection flow

Table 2. **The variation of the recovery factor for crude oil at the end of the re-saturation period in crude oil**

$r = \frac{K_{max}}{K_{min}}$	***The oil viscosities (μ_t, cP)***					
	2	***5***	***10***	***20***	***50***	***100***
3	9,91	8,53	8,02	7,75	7,58	7,52
5	15,11	13,39	12,74	12,39	12,17	12,09
10	23,87	21,64	20,92	20,91	21,22	21,48
20	31,11	29,48	29,04	28,96	29,39	29,71
50	36,78	35,57	35,31	35,36	35,37	35,92
100	39,54	38,5	38,30	38,39	38,64	39,02

Based on the two tables, the values of the parameter *r* were simulated at different viscosities of crude oil (*fig. 5 and 6*).

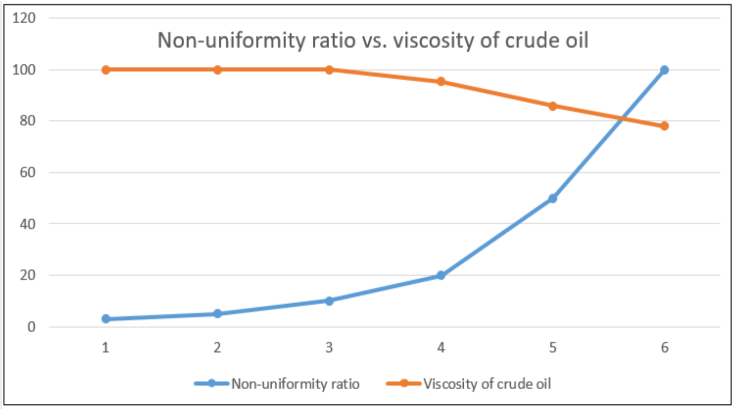

Fig. 5. Variation of the parameter “r” depending on the values of the oil viscosities (various values).

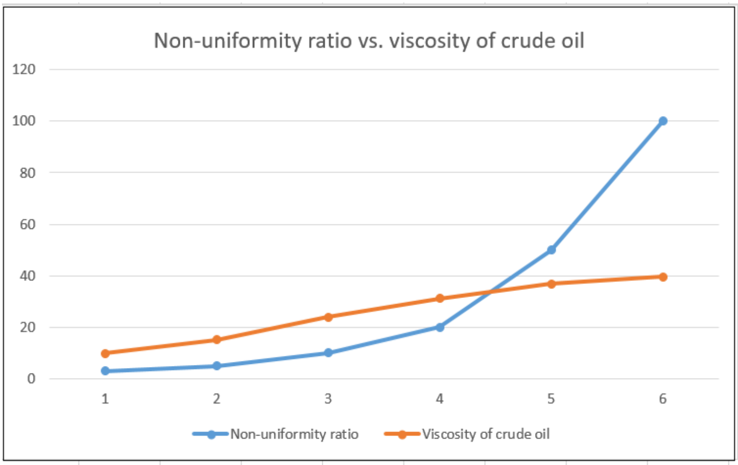

Fig. 6. Variation of the parameter r depending on the values of the viscosities (various values).

4. Conclusions and proposals

In the analysis performed, the recovery factor tends to increase with increasing non-uniformity (for the viscosities of the given crude oil) and of decreasing with increasing viscosity of crude oil for fewer reservoirs. At high non-uniformity ratios, the crude oil recovery factor obtained during the crude oil re-saturation period of the system decreases with increasing viscosity to a point, after which at higher crude oil viscosities it starts to increase again.

This is explained by the distribution of the flow resistances on the component layers of the system, which is easy to understand if we follow the evolution of the injection flow on a uniform layer at ΔP = constant (*fig. 2*).

In conclusion, it can be said that, unlike in the case of uniform reservoirs in which the quantity of crude oil obtained by the end of the re-saturation period represents a negligible percentage compared to the quantity of crude oil, which is subsequently extracted (*fig. 2*). In the case of reservoirs with a high degree of non-uniformity, the recovery factor of the crude oil obtained at the end of the re-saturation period represents important percentages.

It is observed in (*table 2*) that when the water appears even in the reservoirs with a high degree of non-uniformity and high viscosity of crude oil, the gas recovery factor has high values, which allows us to conclude that so far no made many errors if for ease of calculation it was considered that all gases are extracted from the system in the first stage of exploitation of the reservoir (especially for reservoirs with a low degree of non-uniformity).

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