MODELS FOR NONLINEAR DECISIONS OVERVIEW AND 2 CASE EXAMPLES IN FINANCE AND ENERGY

MODELE PENTRU DECIZII NELINIARE SI 2 EXEMPLE DE CAZ IN FINANTE SI ENERGIE

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Abstract: *The latest years evolution in the financial and economic world have proven one more time that the models used to describe behavior of given systems should be extended and the approach on which they are based should change from linear to nonlinear. A number of such models are described in the paper with a synthesis of the main features of nonlinear behavior and a case example is presented on how to describe in order to be able to predict the discontinuous decision associated with the financial crises and with the technological evolution of energy systems. Suggestions are made on the need to control crises not to eliminate them if one wants to better adapt to a nonlinear world dynamic and on the optimal scenarios for the penetration of ALFRED and fuel cell technologies for energy production.*

Keywords: Nonlinear models, discontinuous decisions, financial crises, energy technologies.

Rezumat: Evolutiile din ultimii ani in lumea financiara si economica au dovedit inca o data ca modelele folosite pentru descrierea comportamentului unor sisteme date trebuie sa fie extinse iar abordarea pe care se bazeaza sa se schimbe de la liniar la neliniar. Un numar de asemenea modele sunt descrise in lucrare cu o sinteza a elementelor lor de baza privind comportamentul neliniar si se prezinta exemple de caz privind modul cum este descrisa decizia discontinua asociata cu crizele financiare si cu evolutia tehnologica a sistemelor de energie. Sunt date sugestii privind modul cum se pot controla crizele si nu sa fie eliminate daca se doreste adaptarea la dinamica neliniara a lumii precum si privind scenarii optime de penetrare a tehnologiilor ALFRED si celul de combustibil pentru producerea de energie.

Cuvinte cheie: Modele neliniare, decizii discontinue, crize financiare, tehnologii energetice.

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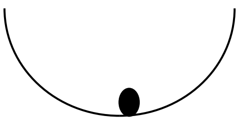
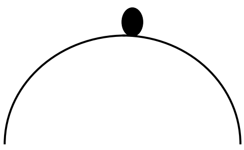
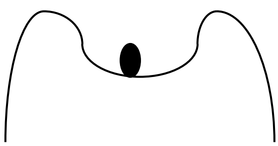
1. Introduction

The development of human infrastructures (energy, transport, communications, etc.) at a planetary level and the capability to work with large amounts of data has revealed the existence of limits both with regard to socio-technological development and with the environmental one. If the limits of the technological development have proved to be of a saturation type i.e. new technologies penetrate to replace the saturated old ones; the environmental evolution shows very clearly that this planet is all we've got. In our strive to master energies at the planetary level, which is not expected to happen in the foreseeable future, we are left with the hope that the environment will be resilient enough to absorb the results of our errors due to a lack of knowledge and of the wish to accept our limits. Several decision reactions are possible:





"whatever you do will change nothing" "anything you do will change everything"



"certain things you do will push the system beyond stable equilibrium, others no"

The perception of change in complex systems and, accordingly, the reaction, show bifurcation like behavior especially when one acquires the conscience of the limits (environmental, technological, social, etc.). The linear mentalities like "whatever you do will change nothing" or "anything you do will change everything" must be changed in more subtle ways of acting that should take into account second order effects which characterise the behavior of the complex systems we are faced with. (e.g. actions like dams building to protect from sea level increase should consider the fact that the production of cement for the dams represents a source of CO_2 which contributes to the raise of sea level.).

During the Middle Ages the indicator of welfare was the quantity of gold one possessed. Accordingly, the "research programmes" of those days were aimed at changing everything to gold. After the occurrence of the energy availability limitations the indicators have changed. Also, the increase in complexity of the interactions among the various systems (energy, population, economy, environment, etc.) has lead to the introduction of aggregated indicators. The planetary view we have today requires the consideration of the meteo geographical conditions for each region and the normalisation of the specific indicators values for making better comparisons. This suggests a personalization of the new technologies for energy supply being implemented in the various regions, which should take into account not only the geographical conditions but also the social ones in order to achieve the maximum efficiency.

Taking decisions for development has always been based on some type of representation of the process. Various models have served as tools to devise or justify decisions. The mathematics behind those models was, usually, linear. Since the behavior of the processes involved was highly non-linear, the approximations made were valid for restricted areas and restricted time intervals. These models were not able to predict the limits beyond which a discontinuous behavior should occur in the systems evolution. Decisions of the type "quit financing a technology and enhance others" are common in the economy. Only in the last years non-linear models, based on non-linear mathematical tools, have made possible the prediction of discontinuous decisions occurring when certain system parameters cross some limit. Although the mathematics involved is more complicated with respect to the linear one, the representation of the systems evolution among limits is more straightforward.

Even if not accepted, limits may, sometimes, be avoided or, seldom, crossed, with the associated shocks. The capability to absorb shocks and still perform normally (resilience) measures the impact of our decisions for development on: the environment, the economy, etc. Accepting the limits, as another alternative, opens the way to understand the mutual interactions among the various systems, becoming, thus, able to change those limits in a sustainable symbiotic evolution.

Negotiating energy versus environment in our development involves information which is not always available, time constants, that may be longer than what we have presently dealt with. The costs and financial measures, implied, may lead, for example, to capital accumulations which we are not prepared to control yet, requiring appropriate administrative structures, or to unusually long pay back times facing the alternative to irreversibly damage the environment. The present changes in the energy generation, transmission, distribution and use systems, leading to more players in the market are raising problems related to the role of a regulator which would avoid the occurrence of chaos in the process, thus, avoiding shocks to the economy. Correlating global change with energy is one of the first projects that considers the interactions among various systems at a planetary scale, opening the way for a closer international co-operation.

The geographical research for fuels together with the scientific research for conversion technologies were one f the main reasons that have lead to the development of infrastructures, such as transportation, telecommunications, etc., which gave us the conscience of, and the possibility to monitor, the influence of our activities on the environment.

Becoming, thus, able to perceive changes, in the complex systems we are interacting with, has influenced our way to understand and, consequently, model more complex behaviors. One should not forget that one of the first class of models which show 'chaotic' behavior was aimed at describing the meteorological behavior (Lorenz, 1957).

2. Development in a limited environment

One of the basic concepts in the non-linear behavior is the limit (separatrix, discontinuity) seen not in the asymptotic way as in the linear theories, but as a drastic change in the behavior of the system. The limits are separating various basins of coherent (possibly linear) behavior. Crossing the limits is the normal way of evolution for the system. As an example, our decisions for development are, frequently enough, to abandon a certain energy program and intensify another, or, which is not symmetric, to intensify a program with respect to others, as in the case of the nuclear energy in Italy and respectively in France. Such decisions may be seen as crossing various types of limits in the 'phase space' of the characteristic parameters of that system.

2.1. Social limits

When talking about social limits we have a spectrum of quantitative and qualitative parameters to consider. Population is the main quantitative one. Actually, it has a double meaning, i.e. first, the purpose of the production in the economic system; second, one of the elements for the production. We must notice that the projections we made assume a certain dynamic for the parameters influencing population evolution. The outcome of fluctuation events like a third world war, or, an epidemic of a virulent unknown disease with no medical technology ready (like the pest in the Middle Ages Europe) are not considered when doing such projections.

On a different line, the economic interactions may be described with models that take into account the geometry of the process. Cities are seen as reactors of economic interactions where the agents are described with neutron physics models. It is shown that the dimension of a city depends on the intensity of the interactions, also that the residents' saving profile occurs as a natural saturation process, and that, specific indicators such as reaction cross sections are explaining in a wider framework the Zipf 'one over income' power law.

Another parameter which, although measurable, has a more qualitative aspect in the sense of describing socio-economic structuring and order, is the energy. Lately, the structuring of energy markets has been ongoing in the economies of the World. This process is shown to have optimality as well as potential generation of chaotic (deterministic chaos) behavior in the penetration of privates on a contestable market, leading to a better explanation of the role of regulatory agencies.

2.2. Technological limits

Creation and penetration of technologies have been regarded from both economic and anthropologic viewpoints. Here too, we encounter the type of nonlinear behavior in the dynamics of technology penetration in a given economy. This penetration of new technologies to replace the saturated old ones is a typical non-linear behavior. The same applies for the polarization of the choice toward one technology although, at the beginning, there co-existed two or more quasi similar technologies in that field. A good example could be the video recording systems which started with both 'Beta' and 'VHS'; after a relatively short time the choice polarized to 'VHS' although the other system was perfectly comparable from the technical point of view; and the process is continuing.

This non-linear behavior in the dynamics of the technologies' evolution shall be considered later in a more formalized way through a qualitative model. New technologies have frequently issued as a reply to the perception of a limit in the development; they sort of refocused the economy on different resources, some of which totally unused to that moment, having even a restructuring effect. Since we have now the perception of reaching limits in the environment a new wave of technologies is supposed to arrive, with a restructuring effect for the economies involved.

2.3. Environmental limits

Although certain limits in local eco-systems have been known for long time, the planetary environment system, yet, too complex for us to assess (based on the relatively scarce amount of data we have till now), the real, short and long term, importance of the limits we have presently identified.

It is certain that there is a strong correlation between the emissions of CO_2 (together with other gases) and the air temperature of the planet. Also, it is ascertained that the integrity of the ozone layer is perturbed by the presence of the CFC's in the atmosphere. But, to what extent the earth may absorb these perturbations still maintaining conditions that are innocuous to humans is a thing that we would not dare try to experience given our present capability to control the whole planet.

3. The perception of changes in complex systems

3.1. Perception of a complex process

The Greenhouse (GH) effect is characterized by a large spectrum of the time constants of the involved processes. Certain economic actions may produce an effect on temperature in decades others, may act continuously and screen the long-time tendency of the evolution. The perception of the interaction such as the one between human society and the planetary system is polarized by its high level of complexity. At one end the interaction may be perceived as "do whatever you please nothing will happen", or, in other words, a totally absorbent (completely resilient) anthropo-planetary system. Whatever big the shock induced to the system by a certain decision, will not produce a loss of equilibrium. A physical representation would be the state point of the system inside an infinitely high characteristic potential well.

At the other end of such a polarization of opinion there is the low resilience perception of the type "anything you do will ruin everything". The physical image of such a case is the system's state point on top of a potential hill; any shock will move it out of the equilibrium position.

In between these two extremes the interaction may be characterized by "certain things you do will push the system beyond stable equilibrium, others no". This involves a finite depth of the characteristic potential well; i.e. certain shocks might push the state point of the system out, either to non-equilibrium positions or, to other equilibrium different from the first.

3.2. From simple to aggregate

The simplest method used for getting an image of the dynamics is to show the evolution of each indicator separately. Although this offers a good way to predict the future values of each indicator, it does not allow foreseeing the global system behavior.

To characterize the interaction, we have to start using combined/aggregated indicators. If, for example, in the hydro-dynamical systems we are very much accustomed with criterial numbers (Reynolds, Prandl, etc.) resulted as a combination of the system parameters (indicators), in the economical systems, although highly dynamic, the normally used characterization of the interactions is done by aggregated indicators resulted from the simple division of only pairs of the simple indicators (e.g. energy/capita, energy/GNP, CO_2 emissions/capita, CO_2 emissions/unit of energy, GNP/capita, etc.). We must notice that the simple division indicators' evolution is easy to verify intuitively, thus, even if there is no model that sustains the interpretation of the indicator related to data, one uses intuition to draw conclusions about the future behaviour of the economic system.

3.3. Rich and Poor trigger second order effects in labour or population migration

The problem under study, nowadays, is the difference of rich versus poor economies' specific behavior. It includes the use of children labor, superseding moral values, as well as migration of population (in and out of various areas) in relation to the economic characteristics (like infrastructure, GNP, etc.) of the respective areas.
Regarding the use of children labor it is shown that this is merely one of the two equilibrium point that result in a bifurcation model on the use of adult labor versus the use of adult and children labor depending on the family subsistence level.

Regarding migration, what is the potential gap that produces a variation of concentration (a flux) of persons from one area to another? With the political transitions we are witnessing we shall consider that people are simply going from poor to reach areas. So, the difference that is intensively perceived between west and east or between north and south is the welfare/poverty one.

Taking into account the welfare/poverty barrier between west and east or north and south and the consequent in-migration of population from poor to reach areas, we may identify some non-linear behavior like the one described below.
The infrastructure measures the efficiency with which an economy makes labour (active population) produce GNP, being expressed as GNP/capita.

Increasing the population, by in-migration, represents an increase of the active population (labor). Over a certain saturation value of the infrastructure's efficiency the increase in population shall be greater than its capacity to produce GNP. So, the GNP/capita will diminish, this being perceived as poverty. Thus, along with in-migration from poor to reach areas there is also an import of poverty into the reach economy.

In parallel, the investments of the reach economies for creating (or developing) infrastructures into either east or south contributes to the increase of efficiency in those economies. Thus, the increase in efficiency will produce a greater GNP/capita, perceived as an import of welfare from the reach economy to the poor one.

If this perception is strong enough the outflow of population might reverse. These reversions may create cycles of in-out migrations from initially poor regions where investments are made in developing the infrastructure. A good enough example is provided by Italy where the out migration of the fifties reversed in mid-seventies this being a sign that the infrastructures were set and operational.

Another typical example is the one of the souths of Italy where, 20 years ago, out-migration was the rule for the workers of that area. Investments done by the government to create infrastructure lead nowadays to a slow-down, if no to a stop, of the out-migration. Based on the ideas above we might expect an in-migration to the south of Italy after a certain number of years. This may seem unbelievable for a 40 (and over) years old Italian, but, it is as unbelievable as the disaggregation of communist like structures may have seemed to an east European 20 years ago.

3.4. New technologies implementation

We are also analyzing the implementation of new technologies into an economy. This will lead to more complex aggregated parameters, but, to a simpler layout of the model which allows a better understanding of the occurrence of discontinuities.

This process has a dynamic characterized by the successive saturation of the old technologies with the emergence of new ones (Kondratiev cycles - A.Grubler, H Nowotny, Towards the fifth Kondratiev Upswing: Elements of an emerging new growth phase and possible development trajectories, IIASA, Nov. 1990). This may be seen as a succession of hysteresis type cycles chained in the three-dimensional space (□, u, v).

The decision of allocating funds between two technologies in competition (or one and the rest of available technologies) is described with a Fokker-Planck equation whose stationary solution leads to a cusp catastrophe. The evolution of the system's trajectory may have a sudden discontinuity associated to the decision to abandon a project.

If one considers the production of technologies along with the production of other goods then it can be demonstrated (Purica, Scientometrics 1987) that a similar relation to the Cobb-Douglas one may model the generation of technologies where, instead of labor, one considers intelligence and, instead of production means, one takes research means.

Moreover, the interplay between the production of GDP and the one of technologies is shown to be ruled by a Henon 'strange attractor'. Also, the gain of information by the experimenter of technologies is measured in a Minkovsky space by an associated Lorenz transform. This is a measure of information resulting from applying a multivalent modal logic and not the typical bivalent one.

4. Decisions for development

4.1. Models for development - linear versus non-linear

Looking at the papers of 20 years ago which tried to forecast development one is stricken by the linear behavior suggested by those results, i.e. up going with various slopes. The first type of forecasting that involved some non-linearities was the resource utilization. In this case after an initial slow increase, due to technology implementation, followed a period of fast increase which ended in a saturation due to the depletion of the resource reservoir. Another field where saturation occurs is the penetration of technologies in the economy. This process was extensively described by Marchetti and Nakicenovich. The results gave the possibility to predict limits, although only saturation ones, which represented a great difference from the former linear models.

Generalisation of these models (Purica, 1979, UN-GPID Project; Ursu, Purica et.al., 1985, Risk Analysis, vol.5, nr.4) have shown that limits are due not merely to saturation but also to strongly non-linear correlation among the systems control parameters, as seen in the example of economic model described above. The interplay between aggregated parameters creates limits in the evolution which include the saturation ones but are not limited to them.

4.2. Avoid or cross the limits - systems resilience

When limits may be predicted one would be tempted, in a first place, to avoid them. On a second thought, after having estimated the shock to the economy, crossing certain limits may become a better decision. Thus, if the systems capability to absorb shocks (resilience) is good enough, then, the decision will have to specify not only where to invest, but also, when, in order to be able to avoid certain limits or/and cross certain preselected others.

The possibility of the non-linear approaches to include the moment of time as an element of the decision is more similar to what we are faced with in day to day life, giving a higher predictability level for these models.

4.3. Sustainability - accepting the limits

Once the consciousness of the system's dynamics is created in a systematic way showing the way to control the evolution trajectory, another possibility occurs, representing an interaction of a superior level. It refers to the fact that by designing the parameters of a system one may control the position and amplitude of the limits. Thus, the alternative of accepting the limits trying to control them by influencing the parameters proves to be the best long run decision. The one word which now, after the issue of the Bruntland Report (Our Common Future), in 1988, is describing this mentality is 'sustainability'. This approach might lead to cope better with the reality of the climate and energy conversion patterns change.

4.4. Indicators of sustainability

At this point „sustainable development" comes in as a fundamentally different approach shifting the focus from economic growth as narrowly constructed in traditional attitudes to economic policy. It speaks of development rather than growth, of the quality of life rather than real incomes alone. That is, sustainable development makes it clear that the very antithesis of growth and the environment is not the issue.

Sustainable development accepts that what we have been calling "economic growth" in the past has been measured by some very misleading indicators. The tendency has been to use a measure of gross national product (GNP) as the basis for economic growth calculations. If GNP increases that is economic growth. But GNP is constructed in a way that tends to divorce it from one of its underlying purposes: to indicate, broadly at least, the living standard of the population. If pollution damages health, and health care expenditures rise, that is an increase in GNP - interpreted as a rise in the "standard of living" - not a decrease. If we use up natural resources then, that is capital depreciation; just as if we have machines, we count their depreciation as a cost to the nation. Yet depreciation on man-made capital is a cost while depreciation of environmental capital is not recorded at all. (D.Pearce, A.Markandya, E.Barbier, Blue Print for a Green Economy, Earthscan Publications Ltd., London, 1990).

The message in the above comments is that, from now on, along with the man-made, material and 'know-how', capital we have to seriously take into account the environmental capital. Finding and implementing indicators to show the depreciation of the later capital will not be an easy task. Implementing environmental standards will certainly bore important costs for all nations involved. These new concepts need to penetrate and spread out into the various economies, such that to become a 'state of mind'. Thus, there is a need to institutionalize them in the broader sense of the word.

The next chapter concentrates on describing the occurrence of institutional structures using the Brusselator model where the reaction diffusion is done with the mimes of Dawkins and showing that institutions occur as Benard-Taylor dynamic stability far from thermodynamic equilibrium, in the mimes' space. The evolution of institutional structures viewed in this prospective reminds of Heraclit's 'panta rhei' 'model' for the world dynamic.

We have been talking about cycles in economy. Considering the product cycles and their superposition into full economy's cycles one may introduce an interpretation based on the fact that the so called 'velocity of rotation of money' is actually a frequency that is correlated to the oscillatory behavior of the economy. Dividing general price into production and transaction prices one may find a conservation law of the monetary mass, similar to energy, over the duration of an economic cycle. The associated equation for this process reminds the well-known Schrödinger equation in quantum physics. It is also shown that along with production functions there may be introduced transition functions for products, these being similar to respectively potential and kinetic energy. Various other findings occur from such considerations related to the discrete (meaning not continuous) nature of economic activity, in a finite resource environment.

Let's start now with an examples in applying decision theory.

4.5. Example 1 - Financial crisis and adaptability

Financial activity is based on credibility and, in its turn, this involves the set up of a collective behavior for which great efforts are made to be kept stable. A loss of credibility can destroy a bank or produce drastic drops in the value of shares as well as create discontinuous changes.

Credibility is a dynamic state. It is acquired through the reaction and diffusion of mind entities called memes that penetrate the minds of people in a collectivity and generate a collective behavior that sometime is very different from the one of the individual. A cultural niche is created associated with each domain of human activity, in this case finance that contains the memes supporting the financial instruments occupying the niche. New instruments (supported by memes) appear and penetrate the niche in a strive for dominating it. The penetration is done to the detriment of other instruments and the process is following a logistic curve.

The logistic curve has two parts in its dynamics. One is an 'exponential growth' when the perception is that there are no limits and the other is the 'saturation' where the limit capacity of the logistic behavior is reached. The change is similar to a phase change from chemical systems, as it was shown considering the characteristics of meme reactions in the process of creative behavior. Thus, the new financial instrument eventually dominates the market. The domination of the market is actually seen in decisions to allocate more resources to the specific instrument. Monitoring of this dynamic in order to determine the change of phase may be done with the derivative of the logistic function. This has a change from increasing to decreasing as the phase switches to saturation. The change may allow to predict in due time the occurrence of the saturation hence, the discontinuous decision.

The decision is represented as a trajectory that evolves into a folded space where there are control parameters, representing the benefits and the risks associated to the financial instrument. The trajectory is controlled by the values of the parameters and since in the decisions space there is a limit of changing behavior given by the folded surface, crossing of that limit by the trajectory, from one fold to the other, is marking the decision to abandon allocating resources to the instrument and divert them to the other instruments in the niche.

This discontinuous decision is perceived as a crisis and there is a changing of the collective behavior that lasts until another new financial instrument penetrates and starts dominating the niche. It seems that there is an asymmetric effect of the ‘exponential growth’ phase in terms of memes, meaning that the collective behavior associated to credibility is restored as if the community had lost memory of the previous crisis cycle. This creates the basis to start a new crisis cycle.

Looking back at the crisis cycle that was described based on the logistic penetration of various memes associated to financial instruments in the banking cultural niche, leading to saturation of the dominant instrument followed by the discontinuous decision to abandon the allocation of resources to that instrument, we end up with the existential question:

4.6. Do we want to eliminate the crises?

Trying to answer it we are coming again at the dichotomy of local versus global view. Locally speaking the crisis cycle is bad for most and good for some – voting it in a democratic way would result to be bad. If one looks at the global level another pattern of behavior is taking shape.

First, we live in a world of change. There is a certain ‘inertia (reluctance) to change’ in human structures and that is manifested in dynamical stability far from thermodynamic equilibrium (Purica 2010).

Second, environment is changing and, in order to adapt, we need, along with resilience, the variability and the capability to change fast (even discontinuously fast sometimes). New discoveries are needed be they in science, in finance, or in fashion, just to mention extremely different domains of human activity. Variability is needed even if only to provide space for new generations’ wish for affirmation and new things. After the crisis, there is change – sudden and with large amplitude – in various economic sectors and even in economic decision behavior. A first reaction is to control things such as not to have them repeated. Although, there is also, as Galbraith pointed out, an effect of short memory, that leads to repeating the main pattern that will generate the next crisis cycle.

This brings to mind a jet fighter that was designed to be inherently instable and its stability controlled by computers – when ever a fast maneuver was needed the instability was let loose and then stability in the new regime was re-controlled back in.

If this approach is applied to economic systems, the crisis cycle provides a way to have a better adaptability to various changes e.g. climate change. One may remember that after the crisis of large car manufacturers in the USA there was increased production of electrical vehicles; this would not have occurred with the same rate if the economy was on an increase path, because of the inertia toward change that we were talking about earlier.

Analyzing the behavior in the financial niche one should understand that no single financial instrument may give the possibility to predict, but the behavior of all instruments should be considered. Every once in a while, one instrument starts to occupy the niche, and that is the moment when the logistic penetration of that instrument should be monitored in order to determine the change of phase from the 'exponential growth' phase to 'saturation'. Obviously, the penetration of one instrument is done to the detriment of another or several others and these have got to be monitored too.

Moreover, there is a decision space where an instrument has a decision trajectory to allocate resources to it during its evolution. This space is having a fold that creates a limit of decision change from allocation of resources to abandoning the specific instrument in favor of others. The decision trajectory in the folded space of the 'cusp' may take several paths and the capability to follow them, either toward a local stability point or to the limit of discontinuous change, allows prediction.

The capability to predict gives the possibility to prepare for the change i.e. to avoid it or, if not possible or not wanted, to be resilient to it. Being capable of doing this may introduce a mechanism of self-feed-back in the evolution of financial systems. This mechanism is not aimed at avoiding crises but at containing their amplitude and or being able to control their frequency to cope with the system's resilience. The dynamics of the financial niche will always generate dominant instruments and saturation – triggering discontinuous decisions – but, having the predictive possibility to control when the limit of change will be approached, allows containing the size of the discontinuity.

It is clear that a connection of the flux of money to other fluxes in the economy is needed. The penetration of energy technologies was shown to be also of the same type (a logistic dynamical technological niche). The penetration of information memes is similar and also labor dynamics should be analyzed. One should also look deeper at the evolution of the fluxes of products to see how the natural resources are brought in to manufacturing centers and resulting products are distributed afterwards; the cycle being closed by waste treatment. Understanding how these niches mutually influence each other is paramount to generate a science of systems of systems dynamics leading to a better set up for a development, where the dynamics of nature balances the dynamics of human structures.

One may see we have come here to the very definition of sustainable development or, in terms of nonlinear science, to a dynamically complex structure of systems, having stable behavior far from equilibrium. The dynamical stability is not about eliminating crisis cycles, but integrating them in a global view of the system behavior. As one may see, on a larger time scale, development is not just exponential growth, but saturation and change too and if we seriously want to understand it, we have to consider the big picture too and get accustomed to having limits. Evolution is not about asymptotically reaching the limits but about crossing them into new basins of behavior.

4.7. Example 2 - Scenarios for the technological evolution model of the power market

In the context of the present situation of the Romanian energy system a number of scenarios were defined that take into consideration the evolution of the ratio between nuclear and renewables (including fuel cells) as the only CO2 emissions free energy production technologies.

Definition of the scenarios is presented in what follows together with a set of preliminary results. The analysis will be extended in the second stage of the project with the consideration of specific details.

The software used for the analysis is described below; they were developed by IAEA (International Atomic Energy Agency) based on the original software from IIASA (International Institute for Applied System Analysis).

• *MAED-to estimate the annual energy (electrical energy and heat) for long term (2011-2070) demand in three possible evolution scenarios of the society:*

– *Slow-pessimist (D1),*

– *Reference medium (D2),*

– *Rapid-optimist (D3).*

• *MESAGE-to generate optimal forecasts for the 3 nuclear scenarios associated with equaly probable options of the political decision factors for the modelling period 2011-2070:*

– *Scenario nuclear low (S1),*

– *Scenario nuclear reference (S2),*

– *Scenario nuclear high (S3).*

The main characteristics of the considered preliminary scenarios are given below, and the preliminary results obtained are presented in the following figures.

SECTOR of NUCLEAR ENERGY:

• S1: only 2 units CANDU (existing + life extension)

• S2: 4 units CANDU – as per national energy strategy 2013-2020 existing and in construction + life extension: also, sensitivity studies for 3 cost scenarios of investment capital

• S3: 4 units CANDU + new capacities NPP installed (gen III + after 2035: PWR/HWR Mix) + gen IV (ALFRED project under development).

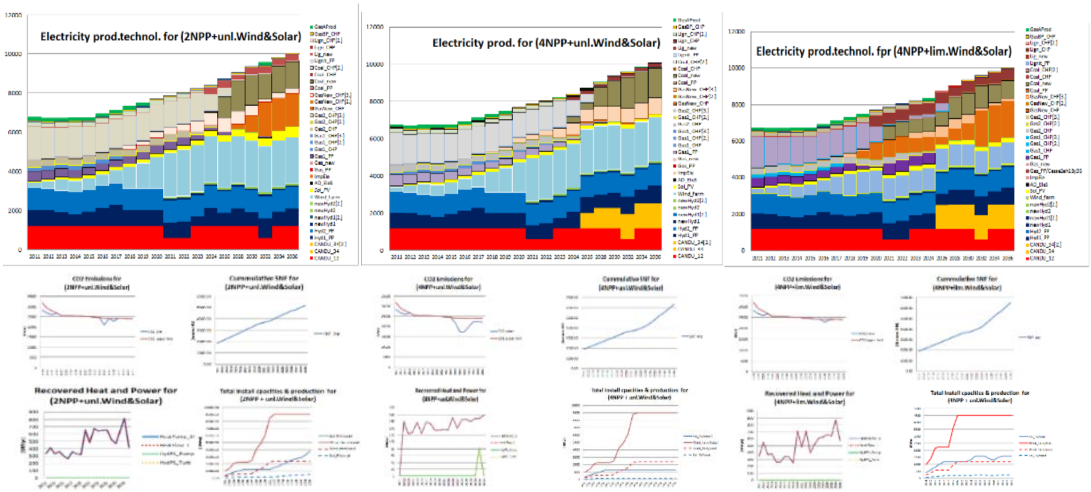
SECTOR HEAT CLASIC & DISTRICT – optimistic hypothesis

• HYDRO (including pump-storage), renewables (wind parks & Solar PV), THERMO (existing/coal, lignite, gas; new/lignite, gas, inclusive heat District-distribution & Storage

UNPROVEN TEHNOLOGIES in the first year of the modelling period:

• They are in development stages and their industrial use will be very costly

• These technologies will be developed and used only in the case where the demand of energy will be greater than the capacity of proven technologies.



Source: authors' contribution

Figure 1. Scenarios for the technological evolution model of the power market

As seen in the figure above the preliminary scenarios generate various evolutions related to the ratio nuclear renewables – that is strongly dependent on the input assumptions of each scenario.

5. Conclusions

Measuring the effects and monitoring this behavior must lead to complex indicators. If in fluid dynamics we have discovered how to represent the system limits of behavior by complex indicators such as the criterial numbers formed of several correlated physical parameters, it seems that we are not at that level, yet, in finance. Although the equations used to build options and derivatives are typical diffusion equations there is not an integrated model unifying the process. We should mention here a profound paper by Kreuser and Siegel (1995) showing that derivative securities instruments may be generated as a combination of atomic instruments having various spectral patterns – similar to the buildup of molecules, from atoms, in chemistry.

One of the conclusions after the 2007 crisis was that there are no models to predict these events. We have tried to look at some models in this paper but it seems that the approach to most of them is based on a linear behavior or a quasi-linear one. There are though several attempts in the literature to escape the narrow vision normally given by linear models. We say 'normally' given because it is typical in the making of a linear model to consider a narrow vicinity around the local point of interest. Obviously, when this point is close to a critical change of phase or a limit of discontinuous behavior, the linear model does not work any longer. One may go to descriptions that synergistically use models from other sciences. An example could be the Bhom's quantum mechanical description that was used to model information dynamics related to given financial systems behavior by Haven (2010), or the description by Purica (2010) of the velocity of rotation of the monetary mass as a typical Schrodinger equation from quantum physics (of course with another meaning of the parameters and variables). There the change is described as the passing of the financial system from one quantum state to another assuming the system may not be in any state, but in well-defined ones similar to quantum states in physics. Absorption or emission of money may be similar to absorption or emission of energy in changing the energy levels of a particle.

One basic conclusion is that we are just beginning to describe the behavior in the financial world and since this involves psychology, evolution, reaction-diffusion, phase change, competition in a specific niche, discontinuous decisions to allocate resources, any attempt to model this behavior should consider all of the above and integrate it in a comprehensive approach. We have tried to do this in a simple (by nonlinear standards) approach with the hope this is somehow the start of a logistic penetration for a different mentality in making 'out of the box' integrated approaches to understanding the dynamics of financial and socio-economic systems' behavior.

Another thing we consider important would be to devise financial behavior that is not based only on an exponential evolution. This would probably involve at least short and long time constants where on short time the behavior is exponential while on long time there may be saturation setting in. In a way, even the time value of money may not be exponential ad infinitum. Moreover, a loan for a very long term may generate memes related to the confidence in the capability to have it paid back that are somehow introducing a given kind of saturation. One may think that the future may bring loans that have a given function of debt service that may be logistic, or have any other time evolution. The financial instruments will be treated not only independently but also in relation to the behavior of all instruments in the niche and we may think of instruments connected to climate change, and other natural processes – weather derivatives are already in place. Development will be disconnected from energy consumption increase and from green house gases emissions, while the indicators of welfare will not only measure growth but also development, understood in a more complex way and 'crises' will be instruments for a faster adaptation of economies to natural and anthropic changes.

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